Key

BIRZEIT UNIVERSITY

Birzeit University
Department of Mathematics
.Math 1321.

First Hour Exam — **Spring 2018**

Name(in Arabic) : .... — **Number:.......**

Instructor of discussion:............ — **Section :......**

Q#1 (60%) circle the most correct answer.

1) The improper integral $\int_0^2 \frac{dx}{\sqrt{4-x^2}}$

a) Diverges

b) Converges to $\frac{1}{2}$

c) Converges to $\frac{\pi}{2}$

d) Converges to $\frac{\pi}{4}$

3) Consider $I_1 = \int_0^1 \frac{dx}{\sqrt{x}}$ and $I_2 = \int_0^1 \frac{dx}{x^2}$ Then

a) Both Integrals Converge

b) Both Integrals Diverge

c) I_1 converges and I_2 diverges

d) I_2 converges and I_1 diverges

3) The Series $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{6}+\frac{1}{8}-\cdots$ converges by Leibnize theorem to L.

If we use $S_3 = 1-\frac{1}{2}+\frac{1}{4}$ to approximate L then

a) $\frac{3}{4} \leq L \leq \frac{11}{12}$

b) $\frac{7}{12} \leq L \leq \frac{3}{4}$

c) $\frac{1}{2} \leq L \leq \frac{3}{4}$

d) $\frac{1}{4} \leq L \leq \frac{1}{2}$

4) The sequence $\{a_n\} = \{1 - \frac{1}{n^2}\}^n$

$a)$ Converges to e^{-1}

$b)$ Converges to e

$c)$ Converges to 1

$d)$ diverges

5) The series $\sum_{1}^{\infty} \frac{2^n 3^n}{n^n}$

$a)$ Converges by n^{th} term test

$b)$ Diverges by Ratio test

$c)$ Converges by Integral test

$d)$ Converges by n^{th} root test

6) One of the following Statements is false

a) if $\sum_{1}^{\infty} |a_n|$ converges then $\sum_{1}^{\infty} a_n$ converges

b) if $\sum_{1}^{\infty} a_n$ and $\sum_{1}^{\infty} b_n$ Both converge Then $\sum_{1}^{\infty} (a_n + b_n)$ Converge

c) If $\lim_{x \to \infty} a_n \neq 0$ Then $\sum_{1}^{\infty} a_n$ Diverges

d) if $\sum_{1}^{\infty} (-1)^n a_n$ Converges Then $\sum_{1}^{\infty} a_n$ Converges

7) The Series $\sum_{n=2}^{\infty} \frac{2}{(2n-3)(2n-1)}$

a) Converges to $\frac{1}{2}$

$b)$ Converges to 0

$c)$ Converges to 1

$d)$ Diverges

8) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$

$a)$ Converges Absolutely

$b)$ Converges conditionally

$c)$ Divergess by alternating series theorem

$d)$ Diverges by nth-term test

9) The sequence $\{ a_n \} = \{ \frac{1-(-1)^n}{n} \}$

$a)$ Converges to 1

$b)$ Converges to 0

$c)$ Converges to 2

$d)$ Diverges

10) $\sum_{1}^{\infty} \frac{2+\sin n}{n}$

$a)$ Converges by Alternating Series Theorem

$b)$ Converges by Integral test

$c)$ Diverges by Direct Comparison Test Compared With $\sum_{1}^{\infty} \frac{1}{n}$

$d)$ Converges by Ratio test

11) $\sum_{1}^{\infty} (\ln(x))^n$ Converges If

$a)$ $-1 < x < 1$

$b)$ $e^{-1} < x < e$

$c)$ $o < x < 1$

$d)$ $0 < x < e$

12) $\sum_{1}^{\infty} \frac{1}{1+\ln(n)}$

$a)$ Converges By Direct Comparison Test Compared With $\sum_{1}^{\infty} \frac{1}{n+1}$

$b)$ Converges By Direct Comparison Test Compared With $\sum_{1}^{\infty} \frac{1}{n^2}$

$c)$ Diverges By Limit Comparison Test Compared With $\sum_{1}^{\infty} \frac{1}{n}$

$d)$ None of the above

13) The Series $\sum_{1}^{\infty} \frac{3^n - 2}{4^n}$

$a)$ Converges to $\frac{3}{2}$

$b)$ Converges to $\frac{3}{4}$

$c)$ Converges to 2

$d)$ Converges to $\frac{7}{3}$

14) The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n(n+1)(n+2)}}$

a) Converge absolutely

b) Converge conditionally

c) Diverges

d) Converges to 2

15) The series $\sum_{1}^{\infty} (-1)^n \left(\frac{n^2+1}{2n^2+1}\right)$

$a)$ Converges by limit comparison test

$b)$ Diverges by n^{th} term test

$c)$ Converges by n^{th} term test

$d)$ Converges absolutely

Question # 2(18%) Test for Convergence

a) $\displaystyle\int_1^{\infty} \frac{dx}{\sqrt{e^{2x}+x}}$ Conv. by D.C.T since (1pt)

(2pts)

$$0 \le \frac{1}{\sqrt{e^{2x}+x}} \le \frac{1}{\sqrt{e^{2x}}} = \frac{1}{e^x}$$

and $\displaystyle\int_1^{\infty} \frac{1}{e^x}\,dx = \lim_{a\to\infty}\int_1^a e^{-x}\,dx = \lim_{a\to\infty} -e^{-x}\Big|_1^a$ (2pts)

$= e^{-1}$ conv.

other sols using L.C.T with $\int e^{-x}dx$ (1pt)

b) $\displaystyle\int_0^1 \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx = \lim_{a\to 1^-}\int_0^a \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$ (2pts)

$u = \sin^{-1}x$

$du = \dfrac{1}{\sqrt{1-x^2}}\,dx$

$$= \lim_{a\to 1^-} \frac{(\sin^{-1}x)^2}{2}\Big|_0^a = \frac{\pi^2}{8} - 0 \quad \text{(2pts)}$$

Conv. (1pt)

c) $\displaystyle\sum_{n=0}^{\infty} \frac{\cos(n\pi)}{n+1} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots$ (2pts)

(2pts)

Conv. by A.S. Th

Question # 3(20%)Consider the power series $\sum_{n=1}^{\infty} \frac{(x-4)^n}{n\,3^n}$

1) Find radius and interval of convergence

2)Find the values of x for which the series

a) converges absolutely b) converges conditionally c) diverges

using n^{th} root test for $\sum |a_n|$

$$\sqrt[n]{|a_n|} = \left(\frac{|x-4|^n}{n \cdot 3^n}\right)^{\frac{1}{n}} = \frac{|x-4|}{n^{\frac{1}{n}} \cdot 3} \longrightarrow \frac{|x-4|}{3}$$

So it converges absolutely for $\frac{|x-4|}{3} < 1$

that is $1 < X < 7$

and diverge for $-\infty < X < 1$ and $7 < X < \infty$.

For $X = 1 \Rightarrow \sum_{1}^{\infty} \frac{(-3)^n}{n 3^n} = \sum_{1}^{\infty} \frac{(-1)^n}{n}$ Conv. Cond.

For $X = 7 \Rightarrow \sum \frac{3^n}{n 3^n} = \sum_{1}^{\infty} \frac{1}{n}$ div.

5pts

So

1) Radius of interval of conv. is 3 5pts

interval of convergence $[1, 7)$. 3pts

2) a) Conv. abs. $(1, 7)$ 3pts

b) conv. cond. $\{1\}$ 3pts

c) div. on $(-\infty, 1) \cup [7, \infty)$ 3pts

Question # 4(6%)

Consider the sequence $a_1 = -1$, $a_{n+1} = \dfrac{a_n + 6}{a_n + 2}$ for n >1

a) Write the first four terms

b)Given that the sequence converges , find its limit.

a) $a_1 = -1$, $a_2 = \dfrac{a_1+6}{a_1+2} = \dfrac{7}{1}$, $a_3 = \dfrac{a_2+6}{a_1+2} = \dfrac{7+6}{7+2} = \dfrac{13}{9}$

$$a_4 = \frac{a_3+6}{a_3+2} = \frac{\frac{13}{9}+6}{\frac{13}{9}+2} = \frac{67}{31}.$$

2pts

b) $\lim\limits_{n\to\infty} a_{n+1} = \dfrac{\lim\limits_{n\to\infty} a_n + 6}{\lim\limits_{n\to\infty} a_n + 2} \Rightarrow L = \dfrac{L+6}{L+2}$

$L^2 + L - 6 = 0$

$(L+3)(L-2) = 0$

$L = -3, 2$

But all [illegible] ...this except the first

2pts

So $L = 2.$ 1pt.